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Mathematical model of the movement of a fighting tracked vehicle

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A – Research concept and design, B – Collection and/or assembly of data, C – Data analysis and interpretation, D – Writing the article, E – Critical revision of the article, F – Final approval of article

Abstract

OBJECTIVES: Development of movement model of fighting tracked vehicle to study oscillatory processes that cause a dynamic load on the driver's workplace and imitate real conditions of fighting tracked vehicle's movement to develop technical requirements for dynamic simulators with the achievement of high degree of their compliance with the real vehicle. Research hypothesis. Use of the improved mobility platform of dynamic simulators, realizing the conditions as close as possible to conditions of driving a real fighting tracked vehicle. **METHODS:** The presented views are the result of empirical research based on the general scheme of forces acting on a fighting tracked vehicle and allow to theoretically estimate the dynamic load of mechanic-driver's workplace. **RESULTS:** In the study, the author developed an improved model of the movement of a fighting tracked vehicle, which describes the spatial movement of its body in motion on the support surface of a complex profile and allows to estimate theoretically the dynamic workload of the driver's workplace, which provides a basic design of a dynamic platform in six degrees of freedom and will provide to develop the requirements for the modernization of dynamic simulators. **CONCLUSIONS:** When performing combat tasks mechanic-driver of FTV is exposed to the effects of spatial movements of different nature. The mechanic-driver during the movement of FTV feels a wide range of influences that are caused by the interaction of the tracked running gear (TRG) with the bearing surface and change the direction of movement of FTV.

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1. Introduction (aim of paper)

V. Geronimus, V. Vasilchenkov, V. Venikov, M. Kirpichev, M. Mamontov, L. Minkevich and others dealt with general issues of similarity theory. A. Babanko, V. Bonder, R. Zakirov, V. Shukshunov and others dealt with the issues of ensuring similarity in training complexes by means of measuring and information systems. Similar studies have also been conducted by D. Brown, I. Goldstein, J. Christensen, J. O’Brien, and others. These authors point to the need for creating of analytical models of processes of real objects during the development of simulators, which in the future must be implemented in the form of hardware and software capable of implementing the necessary types of similarities.

The dynamic loads from the roughness of the support surface and the heterogeneity of its properties affect a fighting tracked vehicle (FTV) when moving and, accordingly, the crew members.

In this case, the vehicle performs oscillations that are irregular and probabilistic. The bearing surface means the roadbed with which the tracked running gear (TRG) interacts. Analytical and experimental study of such processes is based on the theory of random processes, the main provisions of which are widely used in the analysis of the movement of land vehicles.

Another important component of workloads on the vestibular apparatus in the workplace is the load from the FTV power plant.

The large difference between the amplitudes and frequencies of oscillations of the vehicle when moving on the support surface and the frequencies of perturbations caused by the transmission of the vehicle, allows the possibility of separate evaluation of these components.

Oscillations of the vehicle which are caused by its movement on a basic surface of a difficult profile. The basic design of driving simulators is based on the canonical rules of theoretical mechanics, the theory of mechanisms and vehicles, the main of which is to provide six degrees of freedom of the dynamic simulator cabin and specify the necessary types of oscillating motion that occur during real movement of the machine taking into account outside influence.

The oscillating movements of the vehicle body during operation depend on many factors (nature of the road, speed, frequency of starting-off and braking, number of turns, intensity of firing from the weapon placed on the vehicle, the impact of explosive waves on the vehicle, etc.). The torsion bar and tracked conveyors try to dampen all these oscillations.

However, due to the small duration of the impact and the rigidity of the springs selected by the developers of combat vehicles, these oscillations are non-cyclic and aperiodically damped. Moreover, if in the real dynamics of the movement of vehicle all the force impact on its body can be reduced to the main vector F and the main moment M, then in simulation the main moment M can be neglected, because the oscillating motion occurs on a stationary basis and deviations of rotational nature are damped by springs – equivalent of torsion bar and tracked conveyors (Ivankina *et al*., 1977, pp. 33–37) (Lomov, 1971, pp. 45–50).

2. The main material research

Simulation of vehicle oscillations caused by its movement on the support surface of a complex profile.

On the basis of the general scheme of forces presented in Figure 1, we will reveal partial problems of simulation of oscillatory movement of the fighting tracked vehicle.

In accordance with the general scheme of forces (Figure 1) we will form partial calculation schemes, figures 2.

The general dynamic scheme of oscillating motion of the body of a fighting tracked vehicle under the external influence is shown in Figure 2. The dynamic influence is presented in the form of an arbitrary concentrated force Q, which affects the conditional platform 1 simulating the body. The effect of this conditional platform is manifested in the change of its position with velocity V (Wong, 1982, pp. 33–37).

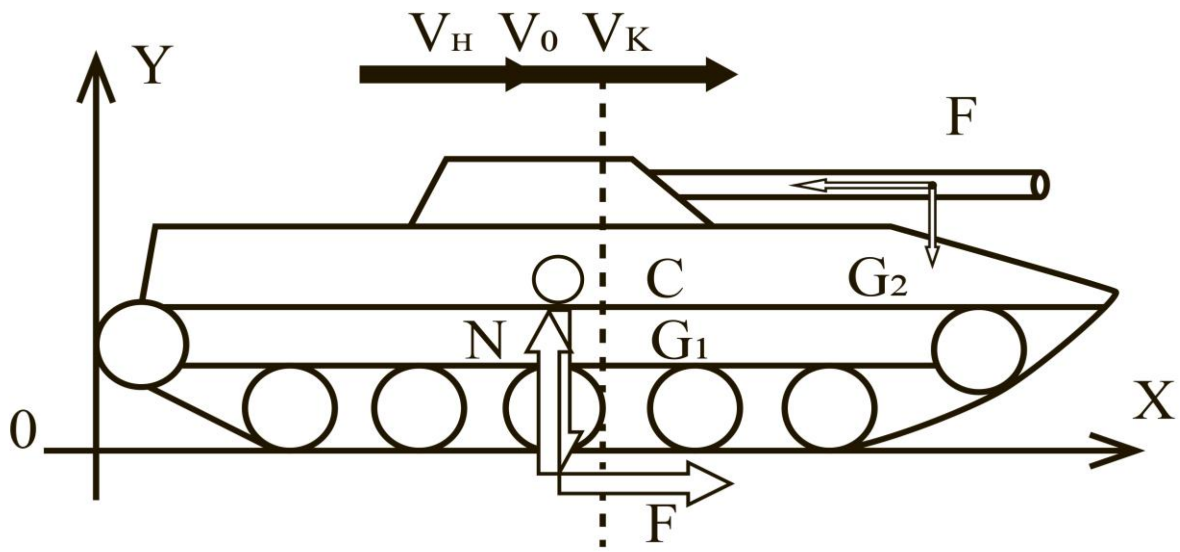


Fig. 1. General scheme of forces acting on FTV

Source: own work.

Based on Figure 1, there are: G1 – gravity of the vehicle, H; N – the reaction of the support surface, H; FT – driving force of friction, H; FB – recoil force, H; G2 – gravity of the projectile, H; VП–

initial speed FTV, m/s; V0 – speed of FTV in the initial position, m/s; VK – final speed of FTV, m/s.

Solving the problem of simulation of oscillating motion of the vehicle from external factors, it can be conditionally taken as an absolutely rigid body, fixed on mutually perpendicular springs of stiffness C (N/m), parallel to which there are damping devices that create braking forces proportional to speed of body movement in each direction. The dynamic effect acting on the body is represented by a constant force Q, which forms the angles α, π – β, π – γ with the coordinate axes (Figure 2).

Let the return forces of the springs be determined from the expressions:

$$\mathrm{F} = c \cdot OM \quad \begin{matrix} Fn = cn\left(Fx = cx : Fy = cy : Fz = cz\right) \\ Fi = \mu Vi\left(F_1 = \mu Vx; F_2 = \mu Vy; F_3 = \mu Vz\right) \end{matrix},$$

where OM – the radius vector of the offset of the point C, m;

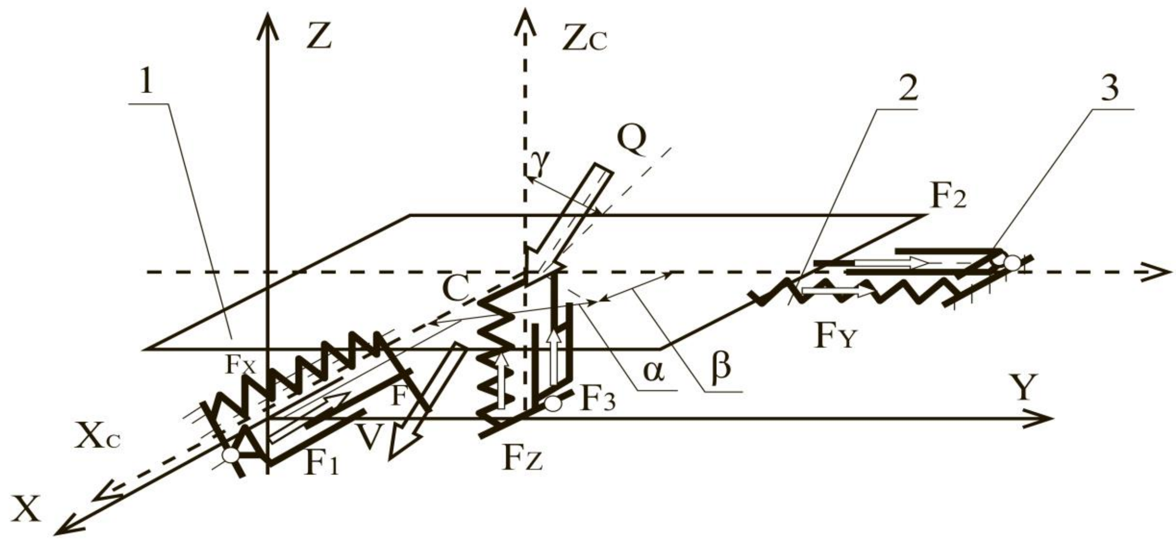
c – spring stiffness, N/m;

μ – the resistance coefficient of the damper, N/m.

Let c and μ be constants in magnitude.

The differential equation of motion of the body along the OX axis will be:

$$m\frac{d^2x}{dt^2} = -cx - \mu Vx + Q\cos\alpha = -cx - \mu\frac{dx}{dt} + Q\cos\alpha \tag{1}$$



1– vehicle body; 2 – mechanical shock absorber; 3 – damping device

Fig. 2. Model of the FTV body suspended on three mutually perpendicular springs under dynamic influence Q

Source: own work.

A linear differential equation of the second order with constant coefficients is obtained. To solve it, we use the Euler's method. Divide both parts of the equation by M. Let's make a permutation of the members of the equation:

$$\frac{d^2x}{dt^2} + \frac{\mu}{m}\cdot\frac{dx}{dt} + \frac{cx}{m} = \frac{Q\cos\alpha}{m} \tag{2}$$

The solution of the differential inhomogeneous equation with respect to x (with the right part consists of the general solution of the homogeneous equation (without the right part) x_1 and the partial solution of the inhomogeneous equation (2) x_2:

$$x = x_1 + x_2. \tag{3}$$

Let's mark the finding of decision for convenience:

$$\frac{\mu}{m} = 2n,\ \frac{c}{m} = k^2,\ \frac{Q}{m} = q, \tag{4}$$

then, equation (2) will take the form:

$$\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + k^2x = q\cos\alpha \tag{5}$$

Write a homogeneous differential equation based on differential equation (5):

$$\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + k^2x = 0. \tag{6}$$

Applying the Euler's substitution, let's find the general solution of equation (6) in the form:

$$x_1 = e^{r\cdot t}, \tag{7}$$

where, r – a constant number.

Find the first and second derivatives of x_1 in time:

$$\frac{dx_1}{dt} = \frac{d}{dt}\left(e^{r\cdot t}\right) = r\cdot e^{r\cdot t}; \frac{d^2x_1}{dt^2} = \frac{d^2}{dt^2}\left(e^{r\cdot t}\right) = r\frac{d}{dt}\left(e^{r\cdot t}\right) = r^2\cdot e^{r\cdot t}.$$

Substituting solution (7) into equation (6), we obtain the characteristic

$$\frac{d^2x_1}{dt^2} + 2n\frac{dx_1}{dt} + k^2x_1 = 0,$$

equation: $r^2e^{rt} + 2nre^{rt} + k^2e^{rt} = 0,$

$$e^{r\cdot t}\left(r^2 + 2nr + k^2\right) = 0. \tag{8}$$

Since $e^{r\cdot t} \neq 0$, the characteristic equation will be:

$$r^2 + 2nr + k^2 = 0,$$
$$D = 4n^2 - 4k^2 = 4\left(n^2 - k^2\right). \tag{9}$$

Find the roots of equation (9):

$$r_{1,2} = \frac{-2n \pm \sqrt{4\left(n^2 - k^2\right)}}{2} = -n \pm \sqrt{n^2 - k^2}. \tag{10}$$

The roots of the equation are obtained

$$r_1 = -n + \sqrt{n^2 - k^2},\ r_2 = -n - \sqrt{n^2 - k^2} \tag{11}$$

Accordingly, the homogeneous differential equation (6) has three solutions:

Search for the laws of oscillating motion of the vehicle's body for 3 cases

The first case of oscillating motion of the vehicle's body is when the resistance of the damper is greater than the stiffness of the spring (n > k)

1) $n > k$. In this case, there will be two real roots r_1 and r_2. Denote the radicand expression as p^2, then:

$$\sqrt{n^2 - k^2} = p, \tag{12}$$

accordingly, the roots of characteristic equation (9) will take the form:

$$r_1 = -n + p,\ r_2 = -n - p \tag{13}$$

Then, the general solution (7) of the homogeneous differential equation (6):

$$x_1 = x_{r1} + x_{r2},$$

where,

$$x_{r1} = e^{r_1 t} = e^{(-n+p)t} = e^{-nt} e^{pt}, \tag{14}$$

$$x_{r2} = e^{r_2 t} = e^{(-n-p)t} = e^{-nt} e^{-pt}, \tag{15}$$

$$x_1 = e^{-nt} e^{pt} + e^{-nt} e^{-pt} = e^{-nt}\left(e^{pt} + e^{-pt}\right).$$

Multiplying x_{r1} and x_{r2} respectively, by arbitrary constants of integrations C_1 and C_2 and adding them, we obtain the general solution of differential equation (6):

$$x_1 = e^{-nt}\left(C_1 e^{pt} + C_2 e^{-pt}\right). \tag{16}$$

Find the constant integrations under the initial conditions:

$$t = 0;\ x = x_0;\ V = V_0.$$

Then $x_0 = e^{-n\cdot 0}\left(C_1 e^{p\cdot 0} + C_2 e^{-p\cdot 0}\right) = C_1 + C_2.$

Because, $x_0 = C_1 + C_2$, then

$$V_0 = -n x_0 + p\left(C_1 - C_2\right).$$

Solve the system of equations:

$$V_0 = \frac{dx_1}{dt} = \frac{d}{dt}\left[e^{-nt}\left(C_1 e^{pt} + C_2 e^{-pt}\right)\right] = \frac{d}{dt}\left(e^{-nt}\right)\left(C_1 e^{pt} + C_2 e^{-pt}\right) +$$
$$+ e^{-nt}\frac{d}{dt}\left(C_1 e^{pt} + C_2 e^{-pt}\right) = -n e^{-nt}\left(C_1 e^{pt} + C_2 e^{-pt}\right) +$$
$$+ e^{-nt}\left[pC_1 e^{pt} - pC_2 e^{-pt}\right] = -n e^{-nt}\left(C_1 e^{pt} + C_2 e^{-pt}\right) + p e^{-nt}\left(C_1 e^{pt} - C_2 e^{-pt}\right) =$$
$$= -n e^{-n\cdot 0}\left(C_1 e^{p\cdot 0} + C_2 e^{-p\cdot 0}\right) + p e^{-n\cdot 0}\left(C_1 e^{p\cdot 0} - C_2 e^{-p\cdot 0}\right) = -n\left(C_1 + C_2\right) + p\left(C_1 - C_2\right).$$

Then:

$$\begin{cases} x_0 = C_1 + C_2 \\ V_0 = -nx_0 + p\left(C_1 - C_2\right) \end{cases}, \begin{cases} C_1 = x_0 - C_2 \\ V_0 = -nx_0 + p\left(x_0 - 2C_2\right) \end{cases},$$

$$\begin{cases} C_1 = x_0 - C_2 \\ 2pC_2 = -V_0 - nx_0 + px_0 \end{cases}, \begin{cases} C_1 = x_0 - \dfrac{x_0\left(p-n\right)-V_0}{2p} \\ C_2 = \dfrac{x_0\left(p-n\right)-V_0}{2p} \end{cases},$$

$$x_1 = e^{-nt}\left(C_1 e^{pt} + C_2 e^{-pt}\right) = e^{-nt}\left(x_0 - \frac{x_0\left(p-n\right)-V_0}{2p}e^{pt} + \frac{x_0\left(p-n\right)-V_0}{2p}e^{-pt}\right) =$$

$$= \frac{x_0\left(p-n\right)-V_0}{2p}e^{-nt}\left(e^{pt} + e^{-pt}\right).$$

We will look for the partial solution of the inhomogeneous differential equation (5) in the form of the right-hand side. Since q and α are constants, we will look for the partial solution (5) in the form

$$x_2 = A = const, \tag{17}$$

Substituting the partial solution (17) into equation (5), and since

$$\frac{dx_2}{dt} = \dot{x}_2 = 0 \quad \text{and} \quad \frac{d^2x_2}{dt^2} = \ddot{x}_2 = 0 \tag{18}$$

we get the identity

$$k^2 A = q\cos\alpha, \tag{19}$$

where,

$$x_2 = A = \frac{q\cos}{k^2}\alpha. \tag{20}$$

Substituting (16) and (20) in (3), we find the general solution of differential equation (5):

$$x = x_1 + x_2 = \frac{x_0\left(p-n\right)-V_0}{2p}e^{-nt}\left(e^{pt} + e^{-pt}\right) + \frac{q}{k^2}\cos\alpha$$

Since the law of motion of the body is expressed as the sum of three non-periodic functions, the resulting motion will be non-periodic.

The second case of oscillating motion of the vehicle's body is when the resistance of the damper is equal to the stiffness of the spring (n = k)

2. $n=k$ Then from formula (10) follows $r_1=r_2=-n$. Partial solutions of the homogeneous differential equation (6) are written in the form:

$$x_{r1} = e^{-nt}, \ x_{r2} = te^{-nt}. \tag{21}$$

The general solution of equation (6) will be

$$x_1 = e^{-nt}\left(C_1 + tC_2\right). \tag{22}$$

Let's define constant integrations under initial conditions: $t=0; x=x_0; V=V_0$. Then:

$$x_0 = e^{-n\cdot 0}\left(C_1 + 0\cdot C_2\right) = C_1.$$

We find V_0:

$$V_0 = \frac{dx_0}{dt} = \frac{d}{dt}\left[e^{-nt}\left(C_1 + tC_2\right)\right] = \frac{de^{-nt}}{dt}\left(C_1 + tC_2\right) + e^{-nt}\left(\frac{dC_1}{dt} + \frac{dtC_2}{dt}\right) =$$
$$= -ne^{-nt}\left(C_1 + tC_2\right) + e^{-nt}C_2 = -ne^{-n\cdot 0}\left(C_1 + 0\cdot C_2\right) + e^{-n\cdot 0}C_2 = -nC_1 + C_2 =$$
$$= -nx_0 + C_2.$$

Where,

$$C_2 = V_0 + nx_0.$$

Then:

$$x_1 = e^{-nt}\left(x_0 + t\left(V_0 + nx_0\right)\right) = x_0\left(2 + nt\right)e^{-nt}.$$

The partial solution x_2 will be determined by formula (20), and then the solution of differential equation (5) will look like:

$$x = x_1 + x_2 = x_0\left(2 + nt\right)e^{-nt} + \frac{q}{k^2}\cos\alpha. \tag{23}$$

Periodic oscillations of the vehicle body in this case, as in the first, will not be, because the law of translational motion of the vehicle body along the axis OX is expressed by the sum of non-periodic functions. Periodic movement will be observed in cases when the force of impact on the vehicle body will be greater than the damping properties of TRG.

The third case of oscillating motion of the vehicle's body is when the resistance of the damper is less than the stiffness of the spring (n > k)

3. $n<k$. In this case, the roots of characteristic equation (9) will be complex:

$$r_{1,2} = -n \pm pi. \tag{24}$$

Then:

$$x_{r1} = e^{(-n+p)t} = e^{-nt}e^{pt};\, x_{r2} = e^{(-n-p)t} = e^{-nt}e^{-pt}.$$

Since $e^{pi} = \cos(pi) + i\sin(pi)$, the solution of the homogeneous differential equation (6) will look like:

$$x_1 = e^{-nt}\left[C_1\cos(pt) + C_2\sin(pt)\right]. \tag{25}$$

Let's search for values of constant integrations under initial conditions:

$$t = 0;\, x = x_0;\, V = V_0.$$

Then,

$$x_0 = e^{-n\cdot 0}\left[C_1\cos(0) + C_2\sin(0)\right] = 1\cdot\left[C_1 + 0\right] = C_1.$$

To find the constant C_2 let's determine the speed of movement of the body:

$$V(t) = \frac{d}{dt}x_1 = \frac{d}{dt}\left(e^{-nt}\left[C_1\cos(pt) + C_2\sin(pt)\right]\right) = \frac{de^{-nt}}{dt}\left[C_1\cos(pt) + C_2\sin(pt)\right] +$$
$$+e^{-nt}\frac{d}{dt}\left[C_1\cos(pt) + C_2\sin(pt)\right] = -ne^{-nt}\left[C_1\cos(pt) + C_2\sin(pt)\right] +$$
$$+e^{-nt}\left[C_2 p\cos(pt) - C_1 p\sin(pt)\right]$$

at $t = 0$

$$V_0 = -ne^0\left[C_1\cos(0) + C_2\sin(0)\right] + e^0\left[C_2 p\cos(0) - C_1 p\sin(0)\right] =$$
$$= -nC_1 = -nx_0 + pC_2.$$

Where,

$$C_2 = \frac{V_0 + nx_0}{p}.$$

Then,

$$x_1 = e^{-nt}\left[x_0\cos(pt) + \frac{V_0 + nx_0}{p}\sin(pt)\right].$$

The partial solution x_2 will be determined by formula (20), and then the solution of differential equation (5) will look like:

$$x = x_1 + x_2 = e^{-nt}\left[x_0\cos(pt) + \frac{V_0 + nx_0}{p}\sin(pt)\right] + \frac{q}{k^2}\cos\alpha.$$

The total law of reaction to the impact will be in the form of damped oscillations.

Characteristic of the circumferential damping oscillation is an expression $\pm\sqrt{C_1^2 + C_2^2}e^{-nt}$. The value n is called the damping factor. The damping decrement is determined by the formula $\delta = e^{-\frac{nT1}{2}}$, where $T1$ is the period of damping oscillations. The nature of the decrement of damped oscillations is presented in Figure 3.

Similar calculations are performed in the direction of the OY and OZ axes, when modeling body oscillations on training models, however, in contrast to real vehicles, where the characteristics c and μ are clearly separated, in models damping devices, in addition to stiffness characteristics c, make noise deviations in the coefficient damper resistance μ.

This is due to the fact that the shock absorbers actually working in the models, in addition to internal friction, give significant mechanical losses at the mounting points. Determining the decrement of the damping oscillations of the FTV hull is the most important characteristic of TRG damping, which determines not only the smoothness of the running, but also the accuracy of firing.

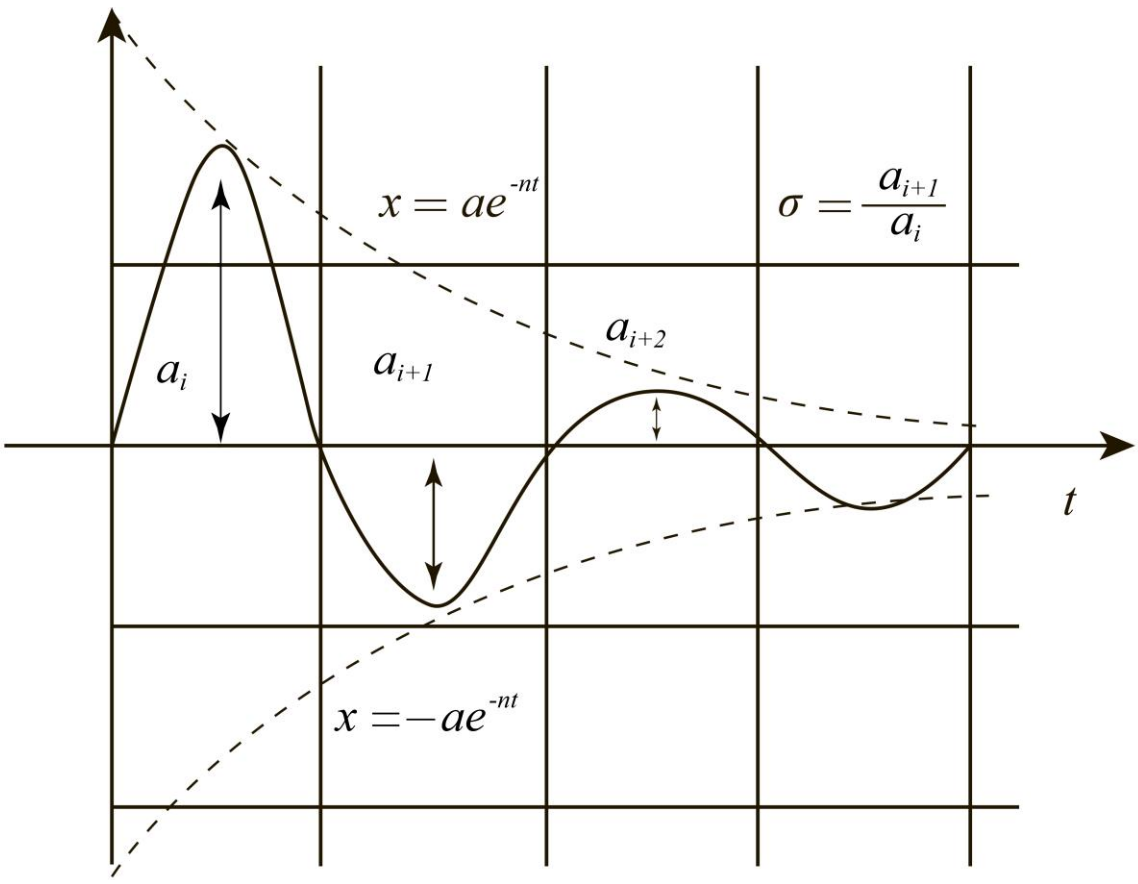


Fig. 3. Amplitude and decrement of damped oscillations

Source: own work.

3. Conclusions

When performing combat tasks mechanic-driver of FTV is exposed to the effects of spatial movements of different nature. The mechanic-driver during the movement of FTV feels a wide range of influences that are caused by the interaction of the tracked running gear (TRG) with the bearing surface and change the direction of movement of FTV.

Existing dynamic simulators reproduce only a partial amount of these effects The main modes of simulation of the movement of the FTV body, which provide the driver with information, the existing mobility platforms are not reproduced in full. These factors are manifested in the insufficient quality of personnel training, and hence low mastery (the degree of realization of potential combat and technical capabilities) of FTV.

One of the ways to solve this problem is to develop a model of the movement of a fighting tracked vehicle, which describes the spatial movement of its body in motion on the support surface of a complex profile that simulates the real conditions of movement of a fighting vehicle. Accordingly, a further direction of research to build the requirements for the latest simulators for driving FTV is to model the oscillations of the operator's workplace and the movement of the simulator cab. Such simulators will provide a higher coefficient of adequacy, and will provide more driving skills compared to existing simulators, and thus increase the level of mastery of FTV and the quality of combat missions.

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