II. TEORETYCZNE I METODOLOGICZNE ZAGADNIENIA BEZPIECZEŃSTWA

PATTERN RECOGNITION METHODOLOGY IN SOLVING OF THE SIMILAR CONFLICTS

Andrzej Ameljańczyk

Military University of Technology

Abstract. The following paper attempts to define the similarity of the conflicts on the example of the selected class of conflict situations, described by the characteristic function. The repository of the conflict patterns, understood as the set of conflicts with stable and easy to obtain solutions has been defined here. The task of the choice of the best conflict solution has been directed to the term of the pattern recognition. The solution method means the take out of the set of the conflict patterns, the conflict which is the most similar to the solved conflict and the use of its solution to deal with the initial conflicts. **Keywords:** similarity of conflicts, metric similarity, graphic similarity, pattern of conflict, ideal games, good solution, pattern recognition, compromise solution, Schmeidler's solution – nucleolus.

1. Introduction

The term of conflict is frequently met in different spheres of life. Obviously, human activity is the widest area of the occurrence of the conflict. It can appear in any situation in which two entities participate. As for as the idea of conflict is dismissed one can consider abstract situations such as "the conflict of parameters" in the process of designing, calculation conflict or the conflict: cost – quality, etc.

All this situations have one common feature which is based on abstract (virtual) or real contradiction of two or more players' businesses. The ability to solve the conflicts, especially group, social, economical, military or even the technical ones means the ability to find the compromise, the conditions of certain balance as well as the stability of entities functioning. There are a lot of different criteria of classification of conflict situations, e.g.:

- the sides number of the conflict;
- the elements number of the strategies sets (the ways of action) of the specified conflict sides ;
- the degree of the disagreement of the aims;
- the possibility of negotiation/ cooperation
- the full set of the information of the conflict sides possibilities.

The branch of science which deals with the formal description (mathematical modeling) of the conflicts as well as the methods of their solutions is the Theory of

Games. Thus, the mathematical model of the conflict is the game [2, 4]. Analogically to the criteria of the classification of real conflict situations, there is a similar classification in the Theory of Games. For the specific classes of the games (conflicts) the adequate methods of mathematical modeling and consequently the adequate models are worked out [2, 4, 6, 10]. They are the object of the studies, mainly in the aspect of the possibilities of the definition and indication of the optimal (the best) solutions in a given conflict situation. Due to the specific aspect of the conflict situations it was impossible to adapt the classical optimizing ideas into the field of the Theory of Games. The leading philosophy of the definition of the best solution of the conflict, is the idea of the balance point [2, 4, 10, 16]. The authors of the main ideas in this field are: von Neumann, Morgenstern, Nash, Owen, Schmeidler, Maschler and Shapley [8, 14, 19]. It is worth mentioning, that the problem of the idea of the point of balance in different publications has won two Nobel prizes.

Particularly, the areas of the applications of the economical, social and collective action have become the subject of a great number of research and analysis and consequently publications [2, 4, 8, 10, 16]. The following paper concerns this problem which in the language of mathematical theory of conflict is named "multiplayer cooperative games". This term refers to the mathematical model of the conflict situation, in which at least two sides of the conflict occur. These sides cannot only negative common strategies (or their fragments) but also decide on a compromise once sharing the profit. Mathematical models of these specified situations "of partial conflict" are presented as the characteristic function [1, 2, 4, 6, 7, 14, 18].

Due to a large number of practical applications, the term of multiplayer cooperative games has achieved extensive theory in the range of defining and indicating the solutions, which should fulfils additional postulates. Subsequently, the postulates of the "perfect solution" ("good solution") include:

- the existence postulate;
- the uniqueness postulate;
- the stability postulate

<u>The postulate of existence</u> concerns the guarantee of the existence of this solution for a possibly big class of conflicts. <u>The postulate of uniqueness</u> signifies the fact of existence of exactly one and only one optimal solution for each conflict. <u>The postulate</u> <u>of stability</u> means that the obtained solution is stable [2, 4, 8]. Therefore, it is not worth changing this solution for any of the potential sides of conflict (of an individual player or any of their coalition).

There are a lot of suggestions of the conflict solution, which can be presented ("modeled") as multiplayer cooperative game. The examples of these concepts include: proportional imputation, solidarity imputation, Shapley's vector, C-core of the game, Schmeidler's solution and compromise solution [2, 4, 6, 13, 16]. In the quoted literature there are different analyses and theorems, which specify the conditions and

possibility of the use of the mentioned above concepts and solutions. As long as the fulfillment of two first postulates of the "perfect solution" is real (most propositions satisfy it), then satisfying of the stability postulate is objectively impossible, since there is "a big class" of the games (conflicts), for which there are not any stable solutions (that means that C-core is an empty set). The solution should be the "closest to the area of stability" when it comes to the stability of final (negotiated) solution.

Generally, the set of all the conflicts (as cooperative games), independently from the number of the elements of the set \mathcal{N} of the players (sides of the conflict) can be divided into two disjoint classes: the set of games with a non-empty C-core of stable solutions and the set of games for which there is not any stable solution $C(v) \neq \emptyset$. In the set of games with non empty C-core there is a partial subset of the games, which has 1-element C-core. As far as the construction of the optimal solution $C(v) = \emptyset$ and negotiation mechanisms are concerned, this class of games is treated as the pattern since each such conflict has "a good solution" resulting from the definition. The set of these games is called the set of "ideal games" or in the other words – the set (repository) of the patterns. The conflict with 1-element C-core is "the happiest conflict" due to the possibility of specifying the optimal (firm and unique) solution [4, 8].

This solution can easily be reached [4, 6, 10] through the solution of a simple system of linear equations [2, 4]. The idea of the concept presented in the following paper is the thesis that the solution of each cooperative game (each conflict) can be provided as solution of "the nearest" (the most similar) game from the set of all games with 1-element C-core [4,6]. Thus, there appears a problem of the meaning of "the nearest" conflict (game), (the most similar conflict). The further part of this paper presents the attempt to define term of "similarity of the conflicts" and in the category of mathematical models of conflicts – the attempt to define the similarity of their models.

2. Multiplayer cooperative games – mathematical models of specific conflict situations

Multiplayer cooperative games are the models of conflict situations, in which it is possible to reach common agreement of the operation strategy as well as negotiations concerning the establishing of coalition and the method of sharing the additional profit by the players out of set $\mathcal{N} = \{1, ..., n, ..., N\}$. The following paper refers to multiplayer cooperative games presented as characteristic function [2, 4, 6]. The key property of the suggested solutions is the so called coalition stability of the solution [4, 6, 8]. The existence of the stable solutions results from the properties of the same conflict, that is the properties of characteristic function $v: 2^{\mathcal{N}} \to \mathcal{R}$. The set of such solutions is C-core of the game [2, 4].

If this set is an empty set, then coalition stable solutions do not exist. Therefore, the specific definition of solution, accepted by the biggest numbers of players, is

needed here. The success of negotiations in reaching the final solution depends to a large extent from the logic of the construction of the suggested definition of the solution (in the range of sharing of the profit). The suggested definition of the solution should definitely fulfil the postulates of the "perfect solution" – that signifies it ought to guarantee the existence of the solution, its uniqueness and correctness in the sense of belonging to C-core, as far as it is non empty. Schmeidler's nucleolus [13] or compromise solution [4,6] are the examples of such solutions. There is only a problem of acceptance of this solution both in the case when $C(v) \neq \emptyset$ as well as in the case, when $C(v) = \emptyset$.

The best case is obviously the situation, when the number of the elements of the set C(v) equals 1. Let $V_*(N)$ means the set of N-person games, for which the number of the elements of the C-core of the game fulfils the condition |C(v)| = 1.

$$V_{*}(N) = \{ v \in V(N) : |C(v)| = 1 \}$$
(2.1)

V(N) – means the set of all N-person games of type $\Gamma = (\mathcal{N}, \mathbf{v})$, for which the characteristic function \mathbf{v} is essential and superadditive [2,4]. The set $V_*(N)$ – is named the set of ideal games (the set of patterns of conflict situations), which as mentioned above, are easy to solve. In the further part of the following paper it will be assumed that one is going to deal with game normalized to $\langle 0,1 \rangle$ -form [2,4,8] due to the fact that each game $\Gamma = (\mathcal{N}, \mathbf{v})$ can be lead to equivalent game $\Gamma = (\mathcal{N}, \mathbf{v})$ in the set of normalized games.

The idea of the suggested method of the solution of the conflict whose model is $\Gamma = (\mathcal{N}, \mathbf{v})$ depends on finding (in the set of patterns $V_*(N)$ such pattern (such ideal game) $\mathbf{v}^* \in V_*(N)$ for which the initial game $\mathbf{v} \in V(N)$ is the most similar. The solution of the ideal game $\mathbf{v}^* \in V_*(N)$ is subsequently assumed as the solution of the initial game $\Gamma = (\mathcal{N}, \mathbf{v})$. Obviously, the necessary condition in the case of games for which $C(\mathbf{v}) \neq \emptyset$ is $C(\mathbf{v}^*) \subset C(\mathbf{v})$. The problem of choice $C(\mathbf{v}^*)$ is trivial, as it leads to the solution of adequate system of linear equations. The set $C(\mathbf{v}^*)$ is indeed 1-element set.

The introduction of this attitude requires the definition of the concept of the similarity of the conflict situations on the basis of the similarity of their models written as cooperative games. This task can be lead to the examination of the similarity of the sequences of numbers with the number of the elements $2^N - N - 2$. It results from the fact that characteristic function of the game $\Gamma = (\mathcal{N}, \mathbf{v})$ can be treated as the element of Euclidean space of the adequate dimension:

$$\mathbf{v} \in \mathcal{R}^{2^N - N - 2} \tag{2.2}$$

For simplification the number $2^N - N - 2$ is symbolized as a_N [2,4]. The general problem of the definition and examination of the objects (the theory of similarity) is widely presented in the literature [6, 10, 20]. In the further part of this paper the definition of the concept of similarity will be provided in reference to the conflicts in the aspect of the similarity of their mathematical models [4, 6].

3. The similarity of multiplayer cooperative games – the similarity of their characteristic functions

It is said, that conflict K_1 is similar to the conflict K_2 if their models $\tilde{A}_1 = (\mathcal{N}, v_1)$ and $\tilde{A}_2 = (\mathcal{N}, v_2)$ are similar. For simplification K_1 is similar to K_2 if v_1 is similar to v_2 .



Drawing 1. The schema of the recognition of the similar conflicts

Definition 3.1 Relation of the similarity of the games is the set R defined as:

$$\boldsymbol{R} = \{ (\boldsymbol{v}_1, \boldsymbol{v}_2) \in \boldsymbol{V}(\boldsymbol{N}) \times \boldsymbol{V}(\boldsymbol{N}) : \boldsymbol{v}_1 \text{ is similar to } \boldsymbol{v}_2 \}$$
(3.1)

$$R \subset V(N) \times V(N)$$

The similarity relations are reflexive and symmetric [2, 4, 6], therefore, bioconditional relations are the specific case of similarity relations [2, 4, 8]. Moreover, as far as a problem of recognition of patterns [6, 8, 10] is concerned the definition of a detection of the similarity is important. This definition allows to compare "the degree of the similarity" of the conflicts. Let $v^* \in V_*(N)$ – be a determined game (characteristic function) – called a pattern.

Definition 3.2

The detection relation of the conflicts similarity is the set $\mathbf{R}^{\nu^*} \subset V(N) \times V(N)$ defined as follows:

$$\boldsymbol{R}^{\boldsymbol{v}^{*}} = \{ (\boldsymbol{v}_{1}, \boldsymbol{v}_{2}) \in \boldsymbol{V}(\boldsymbol{N}) \times \boldsymbol{V}(\boldsymbol{N}) : \text{"function } \boldsymbol{v}_{1} \text{ is more similar to } \boldsymbol{v}^{*} \text{ than funkcjon} \\ \boldsymbol{v}_{2}^{"}$$
(3.2)

In the aspect of the mathematical logic, the sentences " v_1 is more similar to v_2 " and " v_1 is more similar to v^* than function v_2 " have the key meaning. The similarity indicators (similarity function) [6,8] are frequently used in order to define these open sentences in a detailed way.

Definition 3.3

Function $f_p: V(N) \times V(N) \rightarrow \mathbb{R}^n$ is known as function (indicator) of the similarity. The value of the function $f_p(v_1, v_2)$ is interpreted as "the measure (degree) of similarity v_1 to v_2 " [6, 8]. The defined functions (indicator) of the similarity are construed so that they can adopt scalar standardized values from the interval $\langle 0, 1 \rangle$:

$$f_p: V(N) \times V(N) \to 0, 1 \subset \mathcal{R}^1$$
(3.3)

Out of different concepts of the definitions of the similarity (different aspects of the similarity) the following ideas should be mentioned:

Definition 3.4

Two games $\Gamma = (\mathcal{N}, \mathbf{v})$ and $\Gamma = (\mathcal{N}, \mathbf{v})$ are orthogonally similar, if there is $t \in \mathcal{R}$ such as: $\mathbf{v}(S) = \mathbf{v}(S) + t$, $S \in \mathcal{N}(\mathcal{N})$.

Definition 3.5

Two games $\Gamma = (\mathcal{N}, v)$ and $\Gamma = (\mathcal{N}, v)$ are proportionally similar, if there is real number $p \neq 0$ such as v (S) = pv(S), $S \in N(\mathcal{N})$.

Definition 3.6

Two games $\Gamma = (\mathcal{N}, \mathbf{v})$ and $\Gamma = (\mathcal{N}, \mathbf{v})$ are metric similar (ε -metric), if $d_p(\mathbf{v}, \mathbf{v}) \leq \varepsilon$, where:

$$d_{p}(\mathbf{v},\mathbf{v}) = \|\mathbf{v}-\mathbf{v}\|_{p} = \left(\sum_{\boldsymbol{S}\in\mathcal{N}(\mathcal{N})} \left(\left|\mathbf{v}(\boldsymbol{S})-\mathbf{v}(\boldsymbol{S})\right|\right)^{p}\right)^{\frac{1}{p}}, \quad p \ge 1$$
(3.4)

the so called Minkowski's distance of the game v from the game v (2, 4, 6], \mathcal{E} – boundary parameter of the similarity ($\mathcal{E} > 0$) [6],

p - parameter of the distance formula ($p \ge 1$) [4,20].

In the following paper, it is assumed, that p = 2, receiving the geometric (Euclidean) distance [2,8]. An interesting model of the similarity of the conflicts K_1 and K_2 is the similarity of their graphical models [4]. The characteristic function of the conflicts can be presented as the elements (points) of the adequate space (look at 2.2). Web spaces [4] can be used to visualize them. Each sequence of the numbers (but also each characteristic function of the conflict) can be given as a diagram (glyph) in a web space [2]. The degree of their similarity can be indicated by defining the appropriate similarity of the diagram. [4,6,8]. Subsequently, in the case of metric similarity (definition 3.6) the degree of similarity of the conflicts can be defined using the metric distance of the conflicts (3.4)(of the characteristic functions).

Definition 3.7

The indicator (value) of the metric similarity of two conflicts $\Gamma = (\mathcal{N}, \mathbf{v})$ and $\Gamma = (\mathcal{N}, \mathbf{v})$ is the following function:

$$p(\Gamma, \Gamma) = p(\mathbf{v}, \mathbf{v}) = 1 - \alpha_N d(\mathbf{v}, \mathbf{v})$$
(3.5)

where α_N – standardizing coefficient [2,4,8] with the value $\alpha_N = \sqrt{\frac{1}{a_N}}$ while $a_N = 2^N - N - 2$ is a dimension of the space in which the standardized functions \mathbf{v} ' and \mathbf{v} '', (N – the number of players) are defined. For 3-player games $\alpha_3 = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$. It is easily noticed, that if games \mathbf{v} ' and \mathbf{v} '' are "fully similar" (cover in the space of the conflicts), then $d(\mathbf{v}, \mathbf{v}) = 0$ (the distance between them equals zero) and the coefficient of the similarity equals one $p(\Gamma', \Gamma'') = 1$. Whereas $p(\Gamma', \Gamma'') = 0$, when the distance between Γ' and Γ'' is possibly maximal in the whole set V(N). This is the example for $\mathbf{v} = (0,0,0)$ and $\mathbf{v} = (1,1,1)$ or for $\mathbf{v} = (0,1,1)$ as well as $\mathbf{v} = (1,0,0)$. In this cases there is $d(\mathbf{v}, \mathbf{v}') = \sqrt{3}$.

4. The pattern recognition methodology in solving the multiplayer cooperative games

The pattern recognition idea of, today broadly introduced to technical applications, can be successfully used for solving the conflicts. The following method concerns the described above class of conflicts from the area of multiplayer cooperative games. It is assumed that one wants to solve the conflict $\Gamma^{\circ} = (\mathcal{N}, \mathbf{v}^{\circ})$, where \mathcal{N} – the set of players of the game and $\mathbf{v}^{\circ} \in V(N)$. The set V(N) – of all N-person conflicts contains only characteristic functions \mathbf{v} in $\langle 0,1 \rangle$ -reduced form [2, 4, 8].

The set of conflict patterns is the set of ideal games $V_*(N) \subset V(N)$ that is the set of games with 1-element C-core [2,4]. The task of choice of the conflict pattern $\Gamma^* = (\mathcal{N}, \mathbf{v}^*)$ the most similar to the examined (solved) conflict $\Gamma^\circ = (\mathcal{N}, \mathbf{v}^\circ)$ is formed as follows: to take out such characteristic function $\mathbf{v}^* \in V_*(N)$, that:

$$p(\mathbf{v}^*, \mathbf{v}^\circ) = \max_{\mathbf{v} \in \mathcal{V}_*(N)} p(\mathbf{v}, \mathbf{v}^\circ)$$
(4.1)

where function $p(v,v^{\circ})$ is the indicator of metric similarity (3.5). The number $p(v,v^{\circ})$ means the degree of the similarity of the characteristic function of the conflict v to characteristic function v° . The chosen pattern conflict $v^{*} \in V_{*}(N)$ is the most similar conflict out of the set of ideal games to the considered conflict $v^{\circ} \in V(N)$. Its solution turns out to be the solution of the initial conflict $v^{\circ} \in V(N)$.

The papers [2, 4, 8] contain detailed mathematical analysis of the task of maximization (4.1), with the assumption that similarity indicator p is defined on the basis of the geometric distance (3.5). The paper [8] provides full analytic solution of this exercise for 3-player games with the use of algorithm of orthogonal projection of the characteristic function $v \in V(N)$ on the set of ideal games $V_*(N)$ that is on the set of patterns. The obtained results fully proved the thesis, that the methodology of the recognition of the patterns can be used for solving the conflicts. The solution of the pattern conflict v^* appeared to be a proper solution of the initial conflict, fulfilling all the postulates of the so called "good solution" [2, 4, 8]. The 1-element C-core of each ideal game (pattern) is at the same time the Schmeidler's solution, known as the nucleolus (nucleus) of the game [13, 15]. The results of the examination presented in [7, 13] shown that in most cases they are also nucleoluses (nuclei) of the nearest (the most similar) games out of the whole space of the games V(N).

5. Summary

The general indicator of similarity p(v, v) occurs in the formulation of the task of the indication of the most similar pattern (4.1). It is assumed that it is an indicator of metric similarity. The widespread theory deals with the examination

of the different models of similarity. Orthogonal and proportional similarity (as the structural similarity of the conflict) was mentioned in the third point of this paper. The most interesting conclusions can be drawn through the formulation and the examination of the task (4.1) with the use of different (frommetric) models of similarity, such as:

- Tversky's similarity model,
- graphical similarity model, etc

but also with other constructions of the similarity indicators (3.5). Probably, complex algorithm of the pattern recognition, based on the use of the properties of the orthogonal projection can be substituted with much simpler algorithms. In this way it would be practicable to develop the analytical on possibility of the definition of the final solutions out of 3-player games.

LITERATURA:

- 1. A. AMELJAŃCZYK, P. DŁUGOSZ, Computer simulation of a medical diagnosing support process, WAT, Warszawa 2013.
- 2. A. AMELJAŃCZYK, *Matematyczne aspekty modelowania pajęczynowego obiektów*, WAT, Warszawa 2009.
- 3. A. AMELJAŃCZYK, Modelowanie podobieństwa diagnostycznych wzorców medycznych w przestrzeni pajęczynowej, WAT, Warszawa 2011.
- 4. A. AMELJAŃCZYK, Optymalizacja wielokryterialna, WAT, Warszawa 1986.
- 5. A. AMELJAŃCZYK, Podobieństwo modeli graficznych danych medycznych w algorytmach rozpoznawania wzorców diagnostycznych, WAT, Warszawa 2011.
- 6. A. AMELJAŃCZYK, Properties of the Algorithm for Determining an Initial Medical Diagnosis Based on a Two-Criteria Similarity Model, WAT, Warszawa 2011.
- 7. A. AMELJAŃCZYK, Rozwiązania kompromisowe wieloosobowych gier kooperacyjnych *i ich własności*, WAT, Warszawa 2011.
- 8. A. AMELJAŃCZYK, Teoria Gier, WAT, Warszawa 1978.
- 9. A. AMELJAŃCZYK, *Teoria gier i optymalizacja wektorowa*, WAT, Warszawa 1980.
- 10. A. Ameljańczyk, Uproszczona metoda wyznaczania gier równoważnych i ich rozwiązań, WAT, Warszawa 2011.
- 11. S. JIMENEZ, C. BECERRA, Softcardnality-Core: Improving Text Overlap with Distributional Measures for Semantic Textual Similarity, Universidad National de Colombia, 2013.
- 12. W. KRYSICKI, L. WŁODARSKI, Analiza matematyczna w zadaniach, PWN, Warszawa 2011.
- 13. M. LENG, M. PARLAR, *Analytic Solution for the Nucleolus of a Three-Player Cooperative Game*, http://cptra.ln.edu.hk/~mmleng_files/NRL/Nucleolus-100715-Final. pdf?origin=publicationDetail.
- 14. M. MALAWSKI, A. WIECZOREK, H. SOSNOWSKA, Konkurencja i kooperacja. Teoria gier w ekonomii i naukach społecznych, PWN, Warszawa 2006.

- 15. G. OWEN, Teoria gier, PWN, Warszawa 1975.
- 16. B. SIEMIEŃSKA, Metody wyznaczania rozwiązań sytuacji konfliktowych z możliwością kooperacji, WAT, Warszawa 2012.
- 17. B. SIEMIEŃSKA, Badanie podobieństwa konfliktów i ich rozwiązań, WAT, Warszawa 2014.
- 18. T. SMOLEŃ, *Bayesowska teoria oceny podobieństwa. Teoria A. Tversky'ego*, UJ, Kraków, http://ecfi-group.eu/download/papers/17.pdf.
- 19. P.D. STRAFFIN, Teoria gier, Wydawnictwo Naukowe SCHOLAR, Warszawa 2004.
- 20. A. TVERSKY, Features of similarity. Psychological Review, APA, 1977.
- 21. A. TVERSKY, Psychological Review, Hebrew University Jerusalem, Israel 1977.
- 22. http://wms.mat.agh.edu.pl/~zankomar/wyklady/Wyklad8.htm.

METODOLOGIA ROZPOZNAWANIA WZORCÓW W ROZWIĄZYWANIU KONFLIKTÓW PODOBNYCH

Streszczenie. W pracy podjęto próbę formalnego zdefiniowania podobieństwa konfliktów na przykładzie wybranej klasy sytuacji konfliktowych opisanych funkcją charakterystyczną. Zdefiniowano repozytorium wzorców konfliktów, rozumianych jako zbiór konfliktów posiadających stabilne rozwiązania łatwe do uzyskania (wynegocjowania). Zadanie wyznaczenia optymalnego rozwiązania konfliktu sprowadzono do zagadnienia rozpoznawania wzorców. Metoda rozwiązania polega na wyznaczeniu ze zbioru wzorców konfliktów konfliktu najbardziej podobnego do konfliktu rozwiązywanego i wykorzystanie jego rozwiązania do rozwiązania konfliktu wyjściowego.

Słowa kluczowe: podobieństwo konfliktów, podobieństwo metryczne, podobieństwo graficzne, wzorzec konfliktu, rozpoznawanie wzorców, rozwiązanie kompromisowe, rozwiązanie Schmeidlera – c.